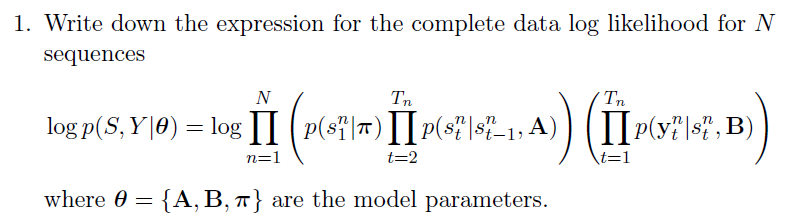
## Complete data log likelihood of the model



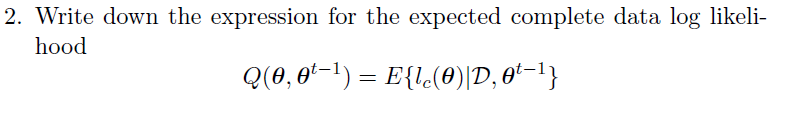
Having N realizations of the HMMs , with , each realization has T time index elements, **,** .

We can obtain the complete data log likelihood of one of these realizations as:

For a group N realizations of the HMM we have, that the complete log likelihood of is:

We finally obtain:

## Expected complete data log likelihood of the model



We know:

Where:

Where:

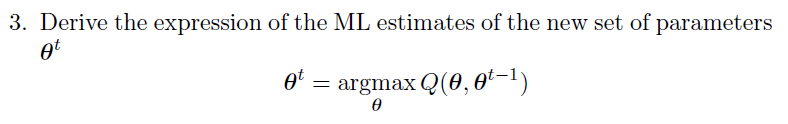
Normalize using the properties:

The alphas and betas are obtained as:

Formula for the alphas:

Formula for the betas:

## Expression for the ML estimates



We know:

Taking the derivative and equaling to 0, we get:

* **Probabilities of the initial state**

We have that:

* **Transition probabilities**

We have that:

* **Values of the parameters of the random distributions b**

Since the observation emission probabilities , follow a D-dimensional multinomial, we use the property of moment matching for the exponential family and we have:

The component of the state is: